B. Tech Degree I & II Semester Examination in Marine Engineering, June 2010

MRE 102 ENGINEERING MATHEMATICS II

Time : 3 Hours | Maximum Marks : 100

I. (a) Find the rank of $A = \begin{bmatrix} 1 & 2 & -4 & 5 \\ -2 & 4 & 1 & 3 \\ 5 & 2 & -13 & 12 \end{bmatrix}$ by reducing to the Echelon form. (5)

(b) Investigate the values of λ and μ so that the equations $2x+3y+5z=9$, $7x+3y-2z=8$, $2x+3y+\lambda z=\mu$ have (i) no solution (ii) a unique solution (iii) an infinite number of solutions. (6)

(c) Verify Cayley – Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and find its inverse. Also express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ as a linear polynomial in A. (9)

OR

II. (a) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin, even though Cauchy – Riemann equations are satisfied at that point. (8)

(b) Evaluate $\int_c \frac{z^2}{z-2}\, dz$ where C is the circle $|z| = 3$. (4)

(c) Find the two Laurent series expansions, in powers of z of the function

$$f(z) = \frac{1}{z(1+z^2)}$$ (8)

III. (a) Solve the following :

(i) $x^4 \frac{dy}{dx} + x^3 y = -\sec xy$

(ii) $(1+y^2)dx = (\tan^{-1} y - x)dy$. (2 x 6 = 12)

(b) Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2} + y = \tan x$. (8)

OR

IV. (a) Solve the following :

(i) $(D-2)^2 y = 8(e^{2x} + \sin 2x + x^2)$

(ii) $\frac{d^2y}{dx^2} - 4y = x \sinh x$. (2 x 6 = 12)

(b) Solve the simultaneous equations :

$\frac{dx}{dt} + 5x - 2y = t$, $\frac{dy}{dt} + 2x + y = 0$ being given $x = y = 0$ when $t = 0$. (8)

(Turn Over)

V. (a) Find a Fourier series to represent $x - x^2$ from $x = -\pi$ to $x = \pi$. (10)

(b) Express $f(x) = \begin{cases} 1 & \text{for } 0 \le x \le \pi \\ 0 & \text{for } x > \pi \end{cases}$ as a Fourier sine integral and hence evaluate $\int_0^\infty \frac{1 - \cos \pi\lambda}{\lambda} \sin x\lambda \, d\lambda$. (10)

OR

VI. (a) Express $f(x) = x$ as a half-range cosine series in $0 < x < 2$. (6)

(b) $\beta(m,n) = \frac{\overline{)m} \times \overline{)n}}{\overline{)m+n}}$. (9)

(c) Evaluate $\int_0^{\pi/2} \sqrt{\tan\theta} \, d\theta$. (5)

VII. Find the Laplace transform of the following :

(i) $e^{-3t}(2\cos 5t - 3\sin 5t)$ (ii) $t^2 \sin at$

(iii) $\frac{1 - e^t}{t}$ (iv) $\left(\sqrt{t} - \frac{1}{\sqrt{t}}\right)^3$ (4 x 5 = 20)

OR

VIII. (a) Find the Laplace transform of the function $f(t) = \begin{cases} \sin wt, & 0 < t < \frac{\pi}{w} \\ 0, & \frac{\pi}{w} < t < \frac{2\pi}{w} \end{cases}$ (10)

(b) Solve by the method of transforms, the equation $y''' + 2y'' - y' - 2y = 0$, given $y(0) = y'(0) = 0$ and $y''(0) = 6$. (10)

IX. (a) Two urns contain respectively 3 black and 2 white balls, 2 black and 3 white balls. One ball is transferred from the second urn to the first and a ball is drawn from the first. What is the probability that it is black? (8)

(b) Evaluate K if the following is a probability density function. Also obtain $P(1 \le x \le 3)$ (6)

X	0	1	2	3
$P(x)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{k}{10}$	$\frac{1}{30}$

A random variable x takes values 1 and 2 with corresponding probabilities $\frac{1}{3}$ and $\frac{2}{3}$.

Find the $E(x)$ and $Var(x)$. (6)

OR

X. (a) Bring out the fallacy in the following "The mean of a binomial distribution is 5 and the SD is 3". (5)

(b) Out of 500 items selected for inspection 0.2% are found to be defective. Find how many lots will contain exactly no defective if there are 1000 lots. (7)

(c) The variable X follows a Normal distribution with mean 45 and S.D 10. Find the probability that $40 < x < 56$. (8)

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